

High-Performance Multi-Coil Inductive Power Transmission Links

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Abstract— Inductive power transmission is widely used to energize implantable microelectronic devices (IMDs), and recharge batteries in mobile electronics and electric cars. Power transfer efficiency (PTE) and power delivered to the load (PDL) are two key parameters in wireless links. To improve the PTE, additional coils have been added to conventional 2-coil links to form 3- and 4-coil inductive links. Several analytical methods for estimating PTE of inductive links have been devised based on circuit and electromagnetic theories. Here, we have analyzed the PTE for multiple capacitively-loaded inductors based on both circuit and coupled-mode theories. We have proven that both methods basically result in the same set of equations in steady state and either method can be applied for short- or mid-range coupling conditions. Through our comprehensive circuit analysis, we have also compared the PTE and PDL of 2-, 3-, and 4-coil links. Our analysis suggests that the 2-coil links are suitable when the coils are strongly coupled and a large PDL is needed. Three-coil links are the best when the coils are loosely coupled, the coupling distance varies considerably, and large PDL is necessary. Finally, 4-coil links are optimal when large PTE is key, the coils are loosely coupled, and their relative distance and alignment are stable.

Index Terms—Inductive Links, Wireless Power Transmission, Resonance Circuits, Power Transfer Efficiency, Near-Field, Coupled-Mode Theory.

I. INTRODUCTION

Inductive power transmission can be used to either continuously power up a device or temporarily recharge its batteries without any direct electrical contact between the energy source and that device. A high efficiency power amplifier (PA), connected to the main energy source on the transmitter side (Tx), often drives the primary coil, which is mutually coupled to a secondary coil on the receiver side (Rx), which is connected to the load (R_L). The load can cover a wide range of applications from high performance and sophisticated implantable microelectronic devices (IMD) with relatively high-power consumption (>100 mW), such as cochlear and retinal implants, to simple and low power radio

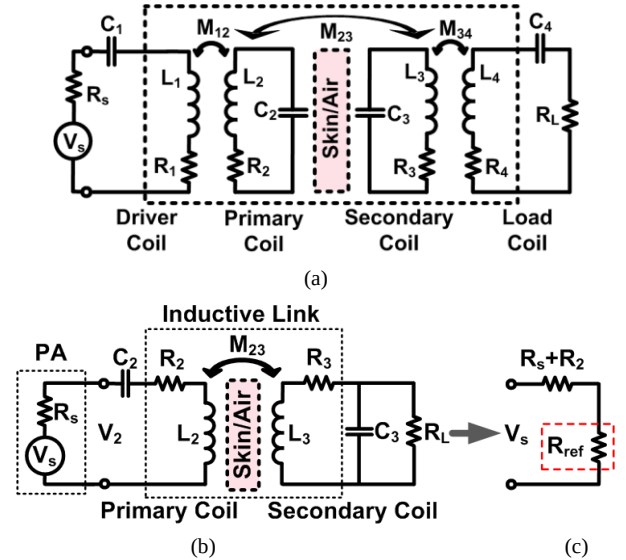


Fig. 1. Lumped circuit models of (a) 4-coil and (b) conventional 2-coil inductive links for wireless power transfer. (c) Equivalent circuit of the reflected load on to the primary loop at resonance.

frequency identification (RFID) tags that cannot use primary batteries due to their cost, size, or lifetime constraints[1]-[6]. The use of this technique to wirelessly transfer energy across a short distance is expected to see an explosive growth over the next decade to cut the power cord in charging mobile electronic devices, operating small home appliances, and energizing electric cars, which have higher power consumptions in the order of hundreds of milliwatts to kilowatts [7]-[12].

The mutual inductance between a pair of coupled coils, M , is proportional to d^{-3} , where d is the center-to-center spacing between the coils when they are in parallel planes and perfectly aligned [5]. A key requirement in all of the above applications is to deliver sufficient power to the load with high power transfer efficiency (PTE) when d is relatively large or the coils are misaligned, i.e. when M is very small (PTE is already large enough for small d). Large PTE is meant to reduce heat dissipation within the coils, tissue exposure to AC magnetic field, which can cause additional heat dissipation in IMDs, size of the main energy source, and interference with nearby electronics to satisfy regulatory requirements [13]-

[15]. At the same time, the link should deliver sufficient power to the load while considering practical limitations of the energy source and the PA. When R_L is constant, PDL would be equivalent to the inductive link voltage gain all the way from the source to the load. Increasing the source voltage, V_s in Fig. 1a, to increase PDL can reduce the driver efficiency, require larger transistors in the PA, and make it more difficult and costly to meet the safety requirements.

Design, theoretical analysis, and geometrical optimization of the conventional 2-coil inductive links have been covered extensively in the literature over the last three decades [16]-[27]. More recently, a 4-coil power transmission link was proposed by physicists in [28] based on coupled-mode theory (CMT) to further increase the PTE, particularly at large d . In the 4-coil arrangement, which schematic diagram is shown in Fig. 1a, a pair of coils is used on the Tx side, which are referred to as the driver, L_1 , and primary, L_2 , coils. A second pair of coils is used on the Rx side, which are referred to as the secondary, L_3 , and load, L_4 , coils. All of these coils are tuned at the same resonance frequency, f_0 , using capacitors $C_1 \sim C_4$. The coils' parasitic resistances are also shown by $R_1 \sim R_4$. The 4-coil method has so far been adopted for transferring power to multiple small receivers, transcutaneous power transmission for $d = 10\sim 20$ mm, and recharging mobile devices in [29], [30], and [31], respectively.

In this paper, we have analyzed multi-coil inductive links in the form of 3- and 4-coil links utilizing both circuit and coupled-mode theories, and compared them with their conventional 2-coil counterpart. Our analyses show 1) circuit and coupled-mode theories both results in the same set of equations for PTE, 2) utilizing the 4-coil method increases the PTE at large d at the cost of a significant reduction in the power delivered to the load (PDL). Therefore, unless the application requires a small amount of power (10s of mW or less), a high driving voltage (V_s) will be required, which can reduce the driver efficiency and lead to safety issues in medical applications. 3) A 3-coil link, initially proposed in [32], not only provides as high PTE as the 4-coil method but also offers a PDL that is significantly higher than both 2- and 4-coil links at large d . In the following section we have constructed a circuit-based theoretical framework to analyze and compare 2-, 3-, and 4-coil inductive links. Section III describes the theory of multi-coil links based on CMT. Measurement results are included in Section IV, followed by concluding remarks in Sections V.

II. CIRCUIT THEORY OF MULTI-COIL INDUCTIVE LINKS

Fig. 1b shows the simplified schematic diagram of a 2-coil inductive link. It is known that the highest PTE across this link can be achieved when both LC tanks are tuned at the same resonance frequency, f_0 [24]. The effect of the Rx side on the Tx side can be modeled at resonance by calculating the reflected impedance,

$$R_{ref} = k_{23}^2 \omega_0 L_2 Q_{3L} = k_{23}^2 (R_s + R_2) Q_2 Q_{3L}, \quad (1)$$

where $k_{23} = M_{23}/\sqrt{L_2 L_3}$, $Q_2 = \omega_0 L_2/(R_s + R_2)$, and $Q_{3L} = Q_3 Q_L/(Q_3 + Q_L)$, in which $Q_3 = \omega_0 L_3/R_3$ and $Q_L = R_L/\omega_0 L_3$ [5]. Q_L is often referred to as the load quality factor, R_s is the loss

associated with the PA, and the value for M_{23} can be calculated from [26]. Hence, the primary loop can be simplified at resonance as the circuit shown in Fig. 1c.

To derive an equation for the PTE at resonance, we should consider that the power provided by V_s divides between $R_s + R_2$ and R_{ref} , and the power delivered to R_{ref} , i.e. the power that is received by the secondary loop, divides between R_3 and R_L , which are the only power consuming components on the Rx side. This will lead to [24]

$$\eta_{2-coil} = \frac{k_{23}^2 Q_2 Q_{3L}}{1 + k_{23}^2 Q_2 Q_{3L}} \cdot \frac{Q_{3L}}{Q_L}. \quad (2)$$

Note that the first and second terms in (2) correspond to the power division between $R_s + R_2$ and R_{ref} , and R_3 and R_L , respectively.

The amount of power delivered to the load (PDL), on the other hand, can be calculated by multiplying the power provided by V_s , i.e. $V_s^2/2(R_s + R_2 + R_{ref})$, by the PTE from (2)

$$P_{L,2-coil} = \frac{V_s^2}{2(R_s + R_2)} \frac{k_{23}^2 Q_2 Q_{3L}}{(1 + k_{23}^2 Q_2 Q_{3L})^2} \cdot \frac{Q_{3L}}{Q_L}. \quad (3)$$

If the simple 2-coil inductive link in Fig. 1b is extended to an m -coil link, in which 1st and m th coils are connected to the energy source and load, respectively, the reflected load from the $(j+1)$ th coil to the j th coil can be found from

$$R_{ref,j,j+1} = k_{j,j+1}^2 \omega_0 L_j Q_{(j+1)L}, \quad j = 1, 2, \dots, m-1 \quad (4)$$

where $k_{j,j+1}$ is the coupling coefficient between the j th and $(j+1)$ th coils and all coils are tuned at the same resonance frequency, f_0 . $Q_{(j+1)L}$ is the loaded quality factor of the $(j+1)$ th coil which can be found from

$$Q_{jL} = \frac{\omega_0 L_j}{R_j + R_{ref,j,j+1}} = \frac{Q_j}{1 + k_{j,j+1}^2 Q_j Q_{(j+1)L}}, \quad j = 1, 2, \dots, m-1 \quad (5)$$

where $Q_j = \omega_0 L_j/R_j$ and R_j are the unloaded quality factor and parasitic series resistance of the j th coil, L_j , respectively. It should be noted that for the last coil, which is connected to the load in series, $Q_{mL} = \omega_0 L_m/(R_m + R_L)$ and for the first coil, which is connected to the source, R_1 also includes the source output impedance, R_s . Therefore, assuming that the coupling between non-neighboring coils is negligible, the PTE from the j th coil to $(j+1)$ th coil can be written as

$$\eta_{j,j+1} = \frac{R_{ref,j,j+1}}{R_j + R_{ref,j,j+1}} = \frac{k_{j,j+1}^2 Q_j Q_{(j+1)L}}{1 + k_{j,j+1}^2 Q_j Q_{(j+1)L}}. \quad (6)$$

Using (4), (5), and (6), the overall PTE in such a multi-coil inductive link can be found from

$$\eta_{m-coil} = \prod_{j=1}^{m-1} \eta_{j,j+1} \cdot \frac{Q_{mL}}{Q_L}, \quad (7)$$

and the PDL from

$$P_{L,m-coil} = \frac{V_s^2}{2(R_s + R_1)} \frac{1}{1 + k_{12}^2 Q_1 Q_{2L}} \eta_{m-coil}. \quad (8)$$

For (7) and (8) to be valid, all coils should be tuned at the same resonance frequency to also achieve the highest PTE and PDL between each neighboring pair of coils [24], and maximize the PTE and PDL of the multi-coil link.

A. Optimal 2-Coil Power Transfer Link

The PTE profile of the 2-coil inductive link according to (2) is a monotonically decreasing function of the coils' coupling distance, d_{23} . However, for a given set of Q_2 , Q_3 and k_{23} values, there is an optimal load, $R_{L,PTE} = \omega_0 L_3 Q_{L,PTE}$, which can maximize the PTE at that particular d_{23} . $Q_{L,PTE}$ can be found by calculating the derivative of (2) vs. Q_L from

$$Q_{L,PTE} = \frac{Q_3}{(1 + k_{23}^2 Q_2 Q_3)^{1/2}}. \quad (9)$$

The maximum PDL at a certain d_{23} can be achieved when the reflected impedance from (1) matches the overall loss of the primary coil, i.e. $R_{ref} = R_s + R_2$ [33]. It should also be noted that in this condition, the PTE is always less than 50%, because half of the power is dissipated in $R_s + R_2$. Thus, the coupling coefficient which maximizes PDL for a certain R_L can be found from,

$$k_{23,PDL} = \frac{1}{(Q_2 Q_{3L})^{1/2}}. \quad (10)$$

Alternatively, by calculating the derivative of (3) vs. Q_L , one can find the optimal load, $R_{L,PDL} = \omega_0 L_3 Q_{L,PDL}$, which can maximize the PDL at a particular d_{23} , where

$$Q_{L,PDL} = \frac{Q_3}{1 + k_{23}^2 Q_2 Q_3}. \quad (11)$$

It is important to note that according to (9) and (11), the maximum PTE and PDL cannot be achieved simultaneously with the same R_L or d_{23} . In the 2-coil links, each of these conditions requires a specific set of k_{23} , Q_2 , and Q_3 , which may not be feasible within the designated constraints. On the other hand, a multi-coil solution provides the designer with more degrees of freedom to optimize the inductive link based on either one of the above requirements. This is the basic idea behind the 3- and 4-coil inductive links, despite their potential negative impact on the size-constrained applications.

B. Three-Coil Power Transfer Inductive Link

The 3-coil inductive link circuit model, which comprises of the primary coil, L_2 , on the Tx side and the secondary and load coils (L_3 and L_4) on the Rx side, has been shown in Fig. 2. If we ignore k_{24} due to large separation between L_2 and L_4 , the PTE of 3-coil link can be found by reflecting R_L back to the primary coil using (4) and calculating PTE from (6) and (7),

$$\eta_{3-coil} = \frac{(k_{23}^2 Q_2 Q_3)(k_{34}^2 Q_3 Q_{4L})}{[(1 + k_{23}^2 Q_2 Q_3 + k_{34}^2 Q_3 Q_{4L})(1 + k_{34}^2 Q_3 Q_{4L})]} \cdot \frac{Q_{4L}}{Q_L} = \eta_{23} \eta_{34} \quad (12)$$

where

$$\eta_{23} = \frac{k_{23}^2 Q_2 Q_{3L}}{1 + k_{23}^2 Q_2 Q_{3L}} = \frac{k_{23}^2 Q_2 Q_3}{1 + k_{23}^2 Q_2 Q_3 + k_{34}^2 Q_3 Q_{4L}},$$

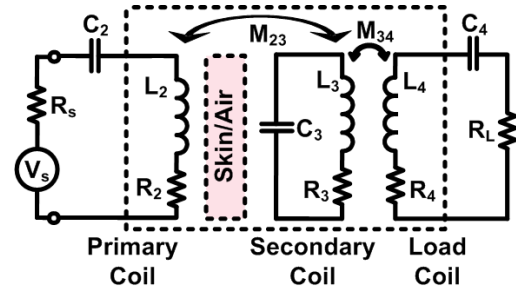


Fig. 2. Lumped circuit model of the 3-coil inductive link.

$$\eta_{34} = \frac{k_{34}^2 Q_3 Q_{4L}}{1 + k_{34}^2 Q_3 Q_{4L}} \cdot \frac{Q_{4L}}{Q_L}. \quad (13)$$

Similarly, the PDL of the 3-coil link can be written as,

$$P_{L,3-coil} = \frac{V_s^2}{2(R_s + R_2)} \frac{(k_{23}^2 Q_2 Q_3)(k_{34}^2 Q_3 Q_{4L})}{(1 + k_{23}^2 Q_2 Q_3 + k_{34}^2 Q_3 Q_{4L})^2} \cdot \frac{Q_{4L}}{Q_L} \quad (14)$$

The problem with the optimal 2-coil power transfer link in section II.A was that for a given set of Q_2 , Q_3 , and k_{23} , the optimal PTE could only be achieved for a certain load, $R_{L,PTE}$, which could be far from the nominal R_L . In the 3-coil power transfer inductive link, however, L_3 , L_4 and M_{34} in Fig. 3 can play the role of an impedance matching circuit (compare with Fig. 1b), which can convert any arbitrary R_L to $R_{L,PTE}$. This is equivalent to having a load quality factor of $Q_L = Q_{L,PTE}$ in the secondary loop of a 2-coil link, which was defined in (9).

This leverage in the design of the 3-coil links has been provided by $k_{34}^2 Q_3 Q_{4L}$ term in (12)-(14). Lowering $k_{34}^2 Q_3 Q_{4L}$ tends to increase η_{23} and at the same time reduces η_{34} in (13), both of which affect the overall PTE in (12). To better understand the PTE variations in a 3-coil inductive link, the effects of k_{34} and d_{23} on the PTE are shown in Fig. 3a for the coils specified in Table-I. The optimal value for $k_{34}^2 Q_3 Q_{4L}$ that maximizes the PTE for a certain d_{23} (or k_{23}) can be found by differentiating (12) with respect to k_{34} ,

$$k_{34,PTE} = \left(\frac{1 + k_{23}^2 Q_2 Q_3}{Q_3^2 Q_{4L}} \right)^{1/4} = \left(\frac{(1 + k_{23}^2 Q_2 Q_3)(1 + R_L / R_4)^2}{Q_3^2 Q_4^2} \right)^{1/4}. \quad (15)$$

For a certain R_L , if the choice of k_{34} in the design of a 3-coil inductive link satisfies (15), then the reflected load onto the secondary loop will satisfy (9) and maximizes the PTE.

Fig. 3b shows the effects of k_{34} and d_{23} on the PDL of the 3-coil inductive link in Table-I, based on (14). It can be seen that there are optimal values for both k_{34} and k_{23} , which can maximize PDL, and in order to find them, (14) should be differentiated with respect to k_{34} and k_{23} ,

$$k_{23,PDL} = \left(\frac{1 + k_{34}^2 Q_3 Q_{4L}}{Q_2 Q_3} \right)^{1/2}, \quad (16)$$

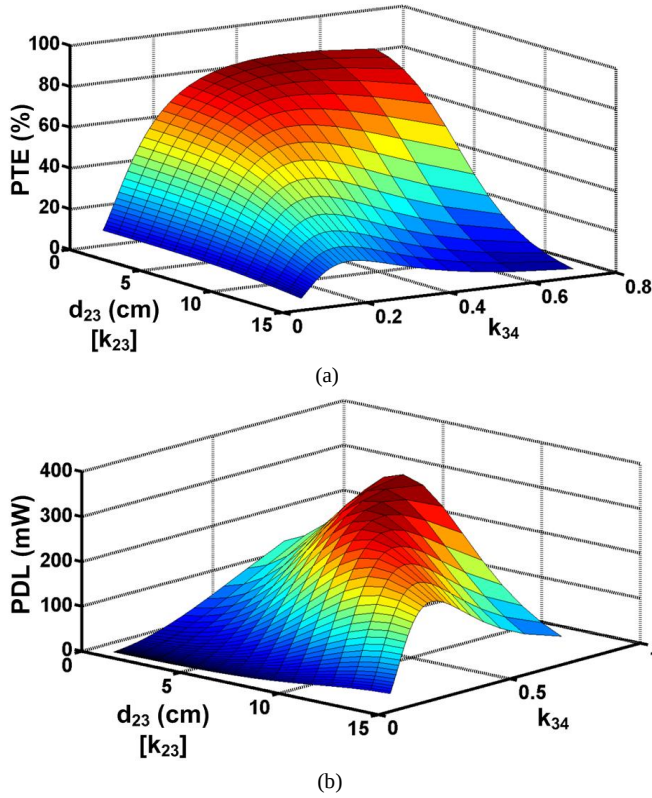


Fig. 3. Simulated (a) PTE and (b) PDL for a 3-coil inductive link as a function of k_{34} and d_{23} for the coils in Table I. $V_s = 1$ V and $R_L = 100 \Omega$.

$$k_{34,PDL} = \left(\frac{1 + k_{23}^2 Q_2 Q_3}{Q_3 Q_{4L}} \right)^{1/2}. \quad (17)$$

These values result in the reflected load on to the primary coil to be equal to R_2 , in order to satisfy (11) for any arbitrary R_L .

C. Four-Coil Power Transfer Inductive Link

The PTE in the 4-coil inductive link, shown in Fig. 1a, can be found by reflecting the resistive components to the left from the load coil back to the driver coil loop, one stage at a time using (4), and calculating the portion of the power that is delivered to the following stage, using (6), until it reaches R_L .

$$\eta_{4-coil} = \frac{(k_{12}^2 Q_1 Q_2)(k_{23}^2 Q_2 Q_3)(k_{34}^2 Q_3 Q_{4L})}{[(1 + k_{12}^2 Q_1 Q_2)(1 + k_{34}^2 Q_3 Q_{4L}) + k_{23}^2 Q_2 Q_3][1 + k_{23}^2 Q_2 Q_3 + k_{34}^2 Q_3 Q_{4L}]} \cdot \frac{Q_{4L}}{Q_L}. \quad (18)$$

To simplify this equation, k_{13} , k_{14} and k_{24} have been neglected in comparison to coupling coefficients between neighboring coils, which are k_{12} , k_{23} and k_{34} [30]. Using the same method, the PDL in the 4-coil inductive link can be found from

$$P_{L,4-coil} = \frac{V_s^2}{2(R_s + R_L)} \frac{(k_{12}^2 Q_1 Q_2)(k_{23}^2 Q_2 Q_3)(k_{34}^2 Q_3 Q_{4L})}{[(1 + k_{12}^2 Q_1 Q_2)(1 + k_{34}^2 Q_3 Q_{4L}) + k_{23}^2 Q_2 Q_3]^2} \cdot \frac{Q_{4L}}{Q_L}. \quad (19)$$

The PTE between loosely coupled primary and secondary coils, L_2 and L_3 , is the dominant factor in determining the overall PTE of the 4-coil link at a large coupling distance, d_{23} . Similar to 3-coil links, when R_L reflects onto L_3 from L_4

according to (4), it reduces the quality factor of L_3 from $Q_3 =$

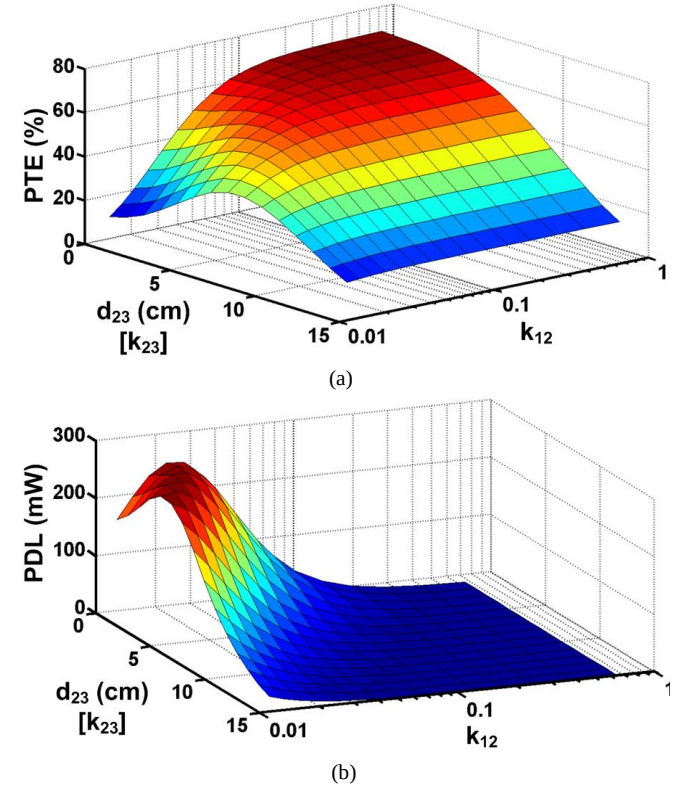


Fig. 4. Simulated (a) power transfer efficiency (PTE) and (b) power delivered to the load (PDL) for a 4-coil inductive link as a function of k_{12} and d_{23} when $k_{34} = 0.22$ for the coils specified in Table I. $V_s = 1$ V and $R_L = 100 \Omega$.

$\omega_0 L_3 / R_3$ to

$$Q_{3L} = \frac{Q_3}{1 + k_{34}^2 Q_3 Q_{4L}}, \quad (20)$$

based on (5). In order to maximize the PTE between L_2 and L_3 , Q_{3L} in (20) should satisfy (9). k_{34} in (20) is, therefore, a key parameter in 4-coil links which allows designers to maximize the PTE for any arbitrary R_L . As mentioned in section II.A, this flexibility is not available in a 2-coil link. Similarly, the total impedance in the secondary coil reflects onto the primary coil, based on (4), and reduces the primary coil's quality factor from $Q_2 = \omega_0 L_2 / (R_s + R_2)$ to

$$Q_{2L} = \frac{Q_2}{1 + k_{23}^2 Q_2 Q_{3L}}. \quad (21)$$

From (21) and (6) it can be inferred that a strong coupling between the primary and secondary coils (i.e. a high k_{23}) reduces Q_{2L} and consequently η_{12} , which is the PTE between L_1 and L_2 . It should also be noted that according to (21), Q_{2L} is roughly proportional to k_{23}^{-2} , where k_{23} is further proportional to d_{23}^{-3} [5]. Therefore, Q_{2L} is proportional to d_{23}^6 , implying that η_{12} will significantly reduce at small d_{23} if k_{12} is not chosen large enough. This effect has been demonstrated in Fig. 4a, which shows the PTE of a 4-coil inductive link as a function of k_{12} and d_{23} for the coils specified in Table I. It can be seen that for small k_{12} , near the origin, the PTE has dropped at short coupling distances due to the small η_{12} . Therefore, small Q_1 and k_{12} will result in a significant drop in η_{12} at small coupling

distances according to (6).

In order to avoid the above problem, k_{12} should be kept large, which according to (4) results in a large reflected load onto L_1 . This can reduce the available power from the source, according to (8), unless V_s is increased. However, large V_s can cause safety issues in medical applications, and this is a major disadvantage of the 4-coil arrangement for inductive power transfer to IMDs, particularly when a high PDL is required.

Fig. 4b shows the PDL from (19) as a function of k_{12} and d_{23} for the coils in Table I. It can be seen that increasing k_{12} results in reducing the PDL when V_s is kept constant. A comparison between Figs. 4a and 4b is instructive by observing that the high PTE and high PDL areas of these surfaces do not overlap, which means that in a 4-coil inductive link there is always a compromise between the highest PTE that can be achieved while delivering sufficient power to the load without surpassing safe V_s limits.

The optimal PTE with respect to d_{23} in a 4-coil link can be found by differentiating (18) in terms of k_{23} , which gives

$$k_{23,PTE} = \left(\frac{\sqrt{1 + k_{12}^2 Q_1 Q_2} \cdot (1 + k_{34}^2 Q_3 Q_{4L})}{Q_2 Q_3} \right)^{1/2}. \quad (22)$$

This equation helps designers to shift the peak of the PTE profile in Fig. 4a towards the nominal coupling distance for certain k_{12} and k_{34} values. Similarly, the optimal PDL, can be found by differentiating (19) in terms of k_{23} , which results in

$$k_{23,PDL} = \left(\frac{(1 + k_{12}^2 Q_1 Q_2) \cdot (1 + k_{34}^2 Q_3 Q_{4L})}{Q_2 Q_3} \right)^{1/2} \quad (23)$$

D. Optimal Multi-coil Link

A comparison between Figs. 2 and 4 reveals a key advantage of the 3-coil links over their 4-coil counterparts, which suffer from poor PDL in areas of the curve that PTE is high (see section II.C). Comparing Figs. 3a and 3b, however, shows that by proper choice of k_{23} and k_{34} , which depend on the coil values and their geometries, designers can establish 3-coil inductive power transfer links that offer both high PTE as well as high PDL. Another advantage of the 3-coil links is that they are not affected by the inefficiency between the driver and primary coils ($\eta_{12} < 1$). However, in the applications that involve small PDL or large R_s , k_{12} helps to decouple R_s from loosely coupled L_2 - L_3 link and, therefore, improve the PTE of the 4-coil link over its 3-coil counterpart.

Fig. 5a compares the 2-coil and 3-coil links' optimal load quality factors, $Q_{L,PTE}$, vs. d_{23} to maximize the PTE for the coils in Table I. Three important points to learn from these curves are: 1) The 2-coil link needs an exceedingly higher $Q_{L,PTE}$ as d_{23} increases, which may not be feasible, particularly in small coils. On the other hand, the 3-coil link satisfies the PTE optimization requirement at various distances with much smaller $Q_{L,PTE}$, which is quite feasible by connecting R_L in series with L_4 as shown in Fig. 2. 2) The optimal $Q_{L,PTE}$ in the 3-coil link is adjustable with k_{34} based on (15), as shown in Fig. 5b, where the optimal PTE has been maintained for the 3-coil link in a wide range of R_L (10 Ω – 1 k Ω) at $d_{23} = 5$ cm. On the other hand, with a 2-coil link the optimal PTE has been

achieved in these conditions only for a specific $R_{L,PTE} = 200 \Omega$

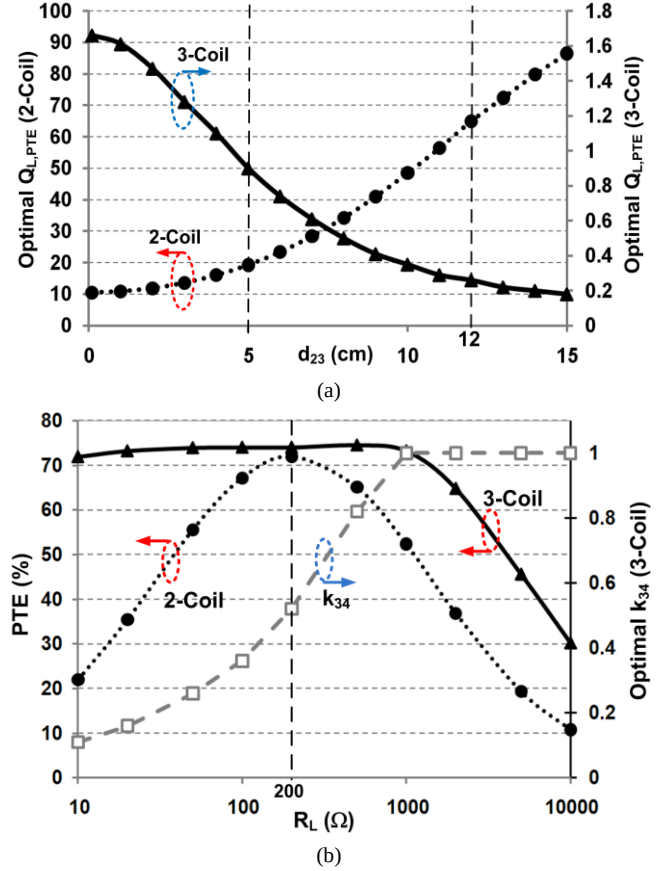


Fig. 5. (a) Optimal load quality factor, $Q_{L,PTE}$, needed to achieve the highest PTE vs. coils' spacing in 2- and 3-coil inductive links ($k_{34} = 0.22$, $R_L = 100 \Omega$, and other parameters from Table-I). (b) k_{34} adjustments based on (15) to maintain the optimal PTE in a 3-coil link vs. R_L at $d_{23} = 5$ cm. The 2-coil link only reaches the optimal PTE for a specific $R_L = 200 \Omega$ that satisfies (9).

that satisfies (9). 3) At small d_{23} , the 2-coil link requires smaller $Q_{L,PTE}$, which is relatively easy to achieve. Therefore, for short distance inductive power transmission, which is the case in most transcutaneous IMD applications, a conventional 2-coil inductive link that is properly designed can be very close to the optimal choice [26], [27].

III. COUPLED-MODE THEORY OF MULTI-COIL LINKS

In this section, we derive the closed-form PTE equations for multi-coil inductive links based on the coupled-mode theory (CMT) and compare them with parallel equations derived from reflected load theory (RLT) in Section II. We prove that both CMT and RLT result in the same set of equations.

CMT Equations for a pair of capacitively-loaded inductors in [32] can be extended to m inductors, in which the 1st and m^{th} inductors are connected to the energy source and load, respectively, as shown in Fig. 6 for $m = 4$ [34]. The time-domain field amplitudes of each inductor, $a_i(t)$, can be expressed as,

$$\frac{da_i(t)}{dt} = -(j\omega + \Gamma_i)a_i(t) + jK_{i-1,i}a_{i-1}(t) + jK_{i,i+1}a_{i+1}(t), \quad i = 2, 3, \dots, m-1 \quad (24)$$

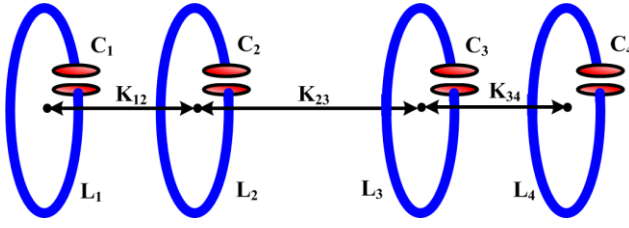


Fig. 6. (a) Four capacitively-loaded, mutually coupled, inductors for wireless power transmission, in which L_1 - K_{12} and L_4 - K_{34} serve as impedance matching elements to improve the PTE.

where $K_{i,i+1}$ and Γ_i are the coupling rate between the i^{th} and $(i+1)^{\text{th}}$ inductor and resonance width of the i^{th} inductor, respectively. For the sake of simplicity, the coupling between non-neighboring inductors has been considered negligible. For the 1st and m^{th} inductors, the field amplitudes are [34],

$$\begin{aligned} \frac{da_1(t)}{dt} &= -(j\omega + \Gamma_1)a_1(t) + jK_{12}a_2(t) + F_s(t) \\ \frac{da_m(t)}{dt} &= -(j\omega + \Gamma_m + \Gamma_L)a_m(t) + jK_{m-1,m}a_{m-1}(t). \end{aligned} \quad (25)$$

In the steady state mode, the field amplitudes in each inductor is considered constant, i.e. $a_i(t) = A_i e^{j\omega t}$. Therefore, the differential equations in (24) and (25) result in a set of $m-1$ equations,

$$\begin{aligned} \Gamma_i A_i - jK_{i-1,i} A_{i-1} - jK_{i,i+1} A_{i+1} &= 0, \quad i = 2, 3, \dots, m-1 \\ (\Gamma_m + \Gamma_L) A_m - jK_{m-1,m} A_{m-1} &= 0. \end{aligned} \quad (26)$$

One can solve (26) to find A_i constants based on the load field amplitude, A_m . From these values, the average power at different nodes of the inductive power transmission link can be calculated. The absorbed power by the i^{th} inductor and the delivered power to R_L can be expressed as $P_i = 2\Gamma_i |A_i|^2$ and $P_L = 2\Gamma_L |A_m|^2$, respectively, from which the total delivered power to the system from source can be found from $P_S = \sum_{i=1}^m P_i + P_L$, using the law of conservation of energy. Finally, the PTE of the m -coil system can be found from [34],

$$\eta_{m\text{-coil}} = \frac{P_L}{P_S} = \frac{\Gamma_L}{\Gamma_m + \Gamma_L + \sum_{i=1}^{m-1} \Gamma_i \left| \frac{A_i}{A_m} \right|^2}. \quad (27)$$

A. Two-Coil Inductive Links

The PTE of the 2-coil system, which only includes L_2 and L_3 in Fig. 6, can be found by simplifying (27) from

$$\eta_{23} = \left[1 + \frac{\Gamma_3}{\Gamma_L} \left(1 + \frac{1}{\text{fom}^2} \left(1 + \frac{\Gamma_L}{\Gamma_3} \right)^2 \right) \right]^{-1}, \quad (28)$$

where $\text{fom} = K_{23} / \sqrt{\Gamma_2 \Gamma_3}$ is the distance-dependent figure-of-merit for energy transmission systems [32]. Resonance widths, $\Gamma_{2,3}$, and coupling rate, K_{23} , are equivalent in terms of circuit model parameters to $\omega/2Q_{2,3}$ and $\omega k_{23}/2$, respectively[32]. Similarly, the load resonance width, Γ_L , is equal to $\omega/2Q_L$. By substituting these in fom and Γ_L / Γ_3 ,

$$\text{fom} = \frac{K_{23}}{\sqrt{\Gamma_2 \Gamma_3}} = \frac{\omega k_{23}}{2} \left(\frac{\omega}{2Q_2} \frac{\omega}{2Q_3} \right)^{-1/2} = k_{23} \sqrt{Q_2 Q_3} \quad (29)$$

$$\frac{\Gamma_L}{\Gamma_3} = \frac{\omega}{2Q_L} / \frac{\omega}{2Q_3} = \frac{Q_3}{Q_L}$$

In the next step, we substitute the CMT parameters from (29) into (28) and recalculate the PTE,

$$\begin{aligned} \eta_{23} &= \frac{1}{1 + \frac{Q_L}{Q_3} \left[1 + \frac{1}{k_{23}^2 Q_2 Q_3} \left(1 + \frac{Q_3}{Q_L} \right)^2 \right]} = \frac{Q_3}{Q_3 + Q_L + \frac{(Q_L + Q_3)^2}{k_{23}^2 Q_2 Q_3 Q_L}} \\ &= \frac{k_{23}^2 Q_2 Q_3}{Q_L + Q_3 + k_{23}^2 Q_2 Q_3 Q_L} \cdot \frac{Q_L Q_3}{Q_L + Q_3}. \end{aligned} \quad (30)$$

After simplification and considering that $Q_{3L} = Q_L Q_3 / (Q_L + Q_3)$, the PTE formula in (30) can be further simplified to,

$$\eta_{23} = \frac{k_{23}^2 Q_2 Q_3 Q_L / (Q_L + Q_3)}{1 + k_{23}^2 Q_2 Q_3 Q_L / (Q_L + Q_3)} \cdot \frac{Q_{3L}}{Q_L} = \frac{k_{23}^2 Q_2 Q_{3L}}{1 + k_{23}^2 Q_2 Q_{3L}} \cdot \frac{Q_{3L}}{Q_L}, \quad (31)$$

which is the same as (2) that was derived via RLT.

B. Three-Coil Inductive Links

The 3-coil inductive link only includes the L_2 - L_3 - L_4 link of the circuit shown in Fig. 6. If we ignore K_{24} due to large separation between L_2 and L_4 , the field amplitudes at each inductor can be calculated by solving a set of two equations in (26), which leads to

$$\frac{A_2}{A_4} = \frac{K_{34}^2 + \Gamma_3(\Gamma_4 + \Gamma_L)}{K_{23} K_{34}}, \quad \frac{A_3}{A_4} = \frac{(\Gamma_4 + \Gamma_L)}{jK_{34}}. \quad (32)$$

PTE of the 3-coil link can then be found by substituting (32) in (27), which leads to (33) after some minor simplifications.

$$\eta_{3\text{-coil}} = \frac{K_{23}^2 K_{34}^2 \Gamma_L}{\Gamma_2 [K_{34}^2 + \Gamma_3(\Gamma_4 + \Gamma_L)]^2 + K_{23}^2 [\Gamma_3(\Gamma_4 + \Gamma_L)]^2 + K_{23}^2 K_{34}^2 (\Gamma_4 + \Gamma_L)} \quad (33)$$

The resonance widths, $\Gamma_{2,4}$, and coupling rates, $K_{23,34}$, in CMT based on circuit parameters are defined as $\omega/2Q_{2,4}$ and $\omega k_{23,34}/2$, respectively [32]. By substituting these parameters in (33) and multiplying both numerator and denominator with $Q_2 Q_3 Q_{4L}^2$, the 3-coil PTE can be found from

$$\begin{aligned} \eta_{3\text{-coil}} &= \frac{k_{23}^2 k_{34}^2 Q_2 Q_3 Q_{4L}^2 / Q_L}{[(k_{34}^2 Q_3 Q_{4L} + 1)^2 + k_{23}^2 Q_2 Q_3 + k_{23}^2 k_{34}^2 Q_2 Q_3 Q_{4L}]} \\ &= \frac{k_{23}^2 k_{34}^2 Q_2 Q_3 Q_{4L}}{[(1 + k_{34}^2 Q_3 Q_{4L})^2 + k_{23}^2 Q_2 Q_3 (1 + k_{34}^2 Q_3 Q_{4L})]} \cdot \frac{Q_{4L}}{Q_L}, \end{aligned} \quad (34)$$

where $1/Q_{4L} = 1/Q_4 + 1/Q_L$. It can be seen that (34), which is derived from the CMT is the same as (12), which is based on the RLT. Therefore, these two formulations are not different in the steady state analysis.

C. Four-Coil Inductive Links

Fig. 6 shows an inductive power transfer link consisting of

four capacitively-loaded inductors, in which L_2 and L_3 are the main coils responsible for power transmission, similar to the 2-coil link, while L_1 and L_4 are added for impedance matching. The field amplitudes at each inductor can be found by solving a set of three equations in (26). A_2 and A_3 can be found based on A_4 from (32) and A_1 can be found from,

$$A_1 / A_4 = - \frac{\Gamma_2 \Gamma_3 (\Gamma_4 + \Gamma_L) + K_{34}^2 \Gamma_2 + K_{23}^2 (\Gamma_4 + \Gamma_L)}{j K_{12} K_{23} K_{34}}. \quad (35)$$

To simplify the analysis, K_{13} , K_{14} and K_{24} have been neglected in comparison to coupling rates between neighboring coils. One can find the 4-coil PTE utilizing CMT by substituting (32) and (35) in (27), which after simplification leads to

$$\eta_{4\text{-coil}} = \frac{K_{12}^2 K_{23}^2 K_{34}^2 \Gamma_L}{D}, \quad (36)$$

where

$$D = \Gamma_1 [\Gamma_2 \Gamma_3 (\Gamma_4 + \Gamma_L) + K_{34}^2 \Gamma_2 + K_{23}^2 (\Gamma_4 + \Gamma_L)]^2 + K_{12}^2 \Gamma_2 [K_{34}^2 + \Gamma_3 (\Gamma_4 + \Gamma_L)]^2 + K_{12}^2 K_{23}^2 \Gamma_3 (\Gamma_4 + \Gamma_L)^2 + K_{12}^2 K_{23}^2 K_{34}^2 (\Gamma_4 + \Gamma_L). \quad (37)$$

We can prove that CMT and RLT equations in (36) and (18) are basically the same by substituting the resonance widths, Γ_{1-4} , and coupling rates, K_{12} , K_{23} , and K_{34} in (36) and (37) with their equivalent circuit parameters, $\omega/2Q_{1-4}$, $\omega k_{12}/2$, $\omega k_{23}/2$, and $\omega k_{34}/2$, respectively [32]. Once both numerator and denominator of (36) are multiplied by $Q_1 Q_2^2 Q_3^2 Q_{4L}^2$, the CMT-based 4-coil PTE leads to,

$$\eta_{4\text{-coil}} = \frac{k_{12}^2 k_{23}^2 k_{34}^2 Q_1 Q_2^2 Q_3^2 Q_{4L}^2 / Q_L}{A^2 + k_{12}^2 Q_1 Q_2 B^2 + k_{12}^2 Q_1 Q_2 k_{23}^2 Q_2 Q_3 + k_{12}^2 Q_1 Q_2 k_{23}^2 Q_2 Q_3 k_{34}^2 Q_3 Q_{4L}} \quad (38)$$

where A and B are

$$A = B + k_{23}^2 Q_2 Q_3, \quad B = 1 + k_{34}^2 Q_3 Q_{4L}. \quad (39)$$

The third and fourth terms of the denominator in (38) can be written in terms of B as,

$$\eta_{4\text{-coil}} = \frac{k_{12}^2 k_{23}^2 k_{34}^2 Q_1 Q_2^2 Q_3^2 Q_{4L}^2 / Q_L}{A^2 + k_{12}^2 Q_1 Q_2 B^2 + k_{12}^2 Q_1 Q_2 k_{23}^2 Q_2 Q_3 B} = \frac{k_{12}^2 k_{23}^2 k_{34}^2 Q_1 Q_2^2 Q_3^2 Q_{4L}^2 / Q_L}{A^2 + k_{12}^2 Q_1 Q_2 B(B + k_{23}^2 Q_2 Q_3)}. \quad (40)$$

By further manipulation of the numerator and denominator,

$$\begin{aligned} \eta_{4\text{-coil}} &= \frac{k_{12}^2 k_{23}^2 k_{34}^2 Q_1 Q_2^2 Q_3^2 Q_{4L}^2 / Q_L}{A^2 + k_{12}^2 Q_1 Q_2 B(A)} = \\ &= \frac{k_{12}^2 Q_1 Q_2 k_{23}^2 Q_2 Q_3 k_{34}^2 Q_3 Q_{4L}}{A(A + k_{12}^2 Q_1 Q_2 B)} \cdot \frac{Q_{4L}}{Q_L} = \\ &= \frac{(k_{12}^2 Q_1 Q_2)(k_{23}^2 Q_2 Q_3)(k_{34}^2 Q_3 Q_{4L})}{A[B(1 + k_{12}^2 Q_1 Q_2) + k_{23}^2 Q_2 Q_3]} \cdot \frac{Q_{4L}}{Q_L}, \end{aligned} \quad (41)$$

TABLE I
INDUCTIVE LINK OPTIMIZED GEOMETRIES

	Parameters	Symbols	2-Coil	3-Coil / 4-Coil
L_1	Inductance (μH)	L_1	-	- / 0.9
	Outer diameter (cm)	D_{o1}	-	- / 16.8
	Fill factor	Φ_1	-	- / 0.13
	Num. of turns	n_1	-	- / 2
	Line width (mm)	w_1	-	- / 8.66
	Line spacing (mm)	s_1	-	- / 2.6
L_2	Quality factor	Q_1	-	- / 255
	Inductance (μH)	L_2	0.9	0.9
	Outer diameter (cm)	D_{o2}	16.8	16.8
	Fill factor	Φ_2	0.13	0.13
	Num. of turns	n_2	2	2
	Line width (mm)	w_2	8.66	8.66
L_3	Line spacing (mm)	s_2	2.6	2.6
	Quality factor	Q_2	255	255
	Inductance (μH)	L_3	0.12	0.4
	Coil diameter (cm)	D_{o3}	4	
	Wire diameter (mm)	w_3	0.64	0.64
	Num. of turns	n_3	1	2
L_4	Line spacing (μm)	s_3	100	100
	Quality factor	Q_3	105.2	177
	Inductance (μH)	L_4	-	0.4
	Coil diameter (cm)	D_{o4}	-	4
	Wire diameter (mm)	w_4	-	0.64
	Num. of turns	n_4	-	2
	Line spacing (μm)	s_4	-	100
	Quality factor	Q_4	-	177
	$L_1 - L_2$ coupling distance (mm)	d_{12}	-	- / 1.5
	$L_2 - L_3$ coupling distance (mm)	d_{23}	120	
	$L_3 - L_4$ coupling distance (mm)	d_{34}	-	9
	Power transfer efficiency (%)	η	15	37 / 35
	Power delivered to load (mW)	PDL	170	260 / 4.4
	Nominal load (Ω)	R_L	100	
	Driver output resistance (Ω)	R_s	0.1	
	Power carrier frequency (MHz)	f_0	13.56	

L_1 and L_2 are hexagonal overlapping PSCs, while L_3 and L_4 are WWCs. Grayed cells indicate the design constraints.

Once A and B are substituted from (39) in (41), it will be identical to (18), which was derived from the RLT. It should be noted that the transient response of inductive links based on circuit and physics theories have been discussed in [34].

IV. SIMULATION AND MEASUREMENT RESULTS

Table I summarizes the results of our optimization procedure in [33] for 2-, 3-, and 4-coil inductive links that operate at $f_0 = 13.56$ MHz and deliver power to a load of $R_L = 100 \Omega$ as efficiently as possible from a nominal coupling distance of $d_{23} = 12$ cm. The coils on the Tx side were considered overlapping hexagonal shaped printed-spiral coils (PSCs), fabricated on cost effective 1.5 mm thick FR4 printed circuit boards (PCB) with 1-oz copper weight (35.6 μm thick), and those on the Rx side were considered wire-wound coils (WWCs) made of magnet wire (enameled copper). The PSC diameter on the Tx side (D_{o1} , D_{o2}) was limited to 16.8 cm due to PCB fabrication constraints. The total weight of the Rx coils was limited to 1.6 g, which relate to the WWC geometries according to [33]. Hence, WWC wire diameter on the Rx side (w_3 , w_4) was limited to 0.64 mm (AWG-22).

To accurately measure the PTE and PDL of multi-coil links, resonance capacitors and R_L are connected to the primary and load coils, which are then considered a complete 2-port system along with the multi-coil inductive link, as shown in Fig. 7.

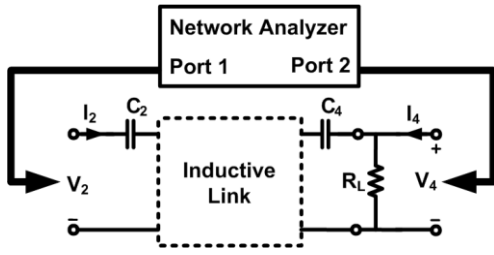


Fig. 7. PTE measurement setups for inductive links using network analyzer with all the coils tuned at the carrier frequency and R_L connected.

The network analyzer is then used to measure the S-parameters, and consequently the Z-parameters are derived. PTE and PDL are found from 2-port equations,

$$PTE = \frac{|Z_{42}|^2}{R_L |Z_{22}| \cos(\angle Z_{22})}, \quad (42)$$

$$PDL = \frac{|V_4|^2}{2R_L} = \frac{|V_2|^2}{2R_L |Z_{22}|^2} |Z_{42}|^2,$$

where $Z_{22} = V_2/I_2$ and $Z_{42} = V_4/I_2$ are derived when $I_4 = 0$. The $I_4 = 0$ requirement in calculating Z-parameters ensures that the network analyzer loading (often 50Ω) on the inductive link does not affect the results. In this method, as the network analyzer sweeps a certain frequency range that includes f_0 , actual power transfer does take place in the form of a small signal injected from Port-1 of the network analyzer to R_L .

Figs. 8a and 9a show the experimental setup for measuring the PTE and PDL of the 3- and 4-coil inductive links, respectively. These coils were fabricated based on the values listed in the Table I, and held in parallel and perfectly aligned using non-conducting Plexiglas sheets and plastic screws to prevent power loss due to eddy currents. Figs. 8b and 9b show 3-D models of the same coils constructed in the HFSS electromagnetic field simulator for 3- and 4-coil links, respectively. In the 4-coil setup, k_{12} was adjusted for a fixed $d_{12} = 1.5$ mm, by changing the amount of overlapping between similar L_1 and L_2 (see Fig. 9). L_3 and L_4 were also similar and provided $k_{34} = 0.22$ at $d_{34} = 9$ mm.

Figs. 10a and 10b compare the measured, simulated (via HFSS), and calculated values of the PTE and PDL, respectively, vs. coupling distance, d_{23} , in 2-, 3- and 4-coil inductive links for $R_s = 0.1 \Omega$. The curves labeled as “Meas1” show the measurement results according to the method proposed in Fig. 7, using a ZVB4 network analyzer (R&S, Germany). It can be seen that these results are in very good agreement with HFSS simulation and calculation results, labeled “Sim” and “Calc”, respectively. As an alternative, we also measured the PTE and PDL of the inductive links using a class-D PA with known power efficiency ($\eta_{PA} = 30\%$) for 2- and 3-coil links in three different distances of 4, 8, and 12 cm, which are labeled as “Meas2” in Fig 10. In this case the measurement results are probably less accurate because of the parasitic components added by the probes for measuring input/output power levels and the fact that the PA’s power

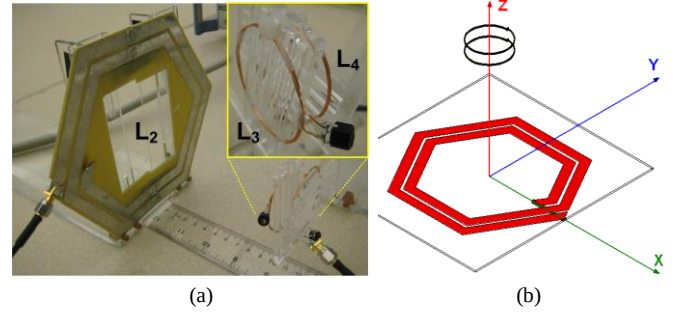


Fig. 8. (a) Experimental setup for measuring the PTE and PDL in a 3-coil inductive link. (b) 3-coil inductive link model in the HFSS. Coil specifications are listed in Table I.

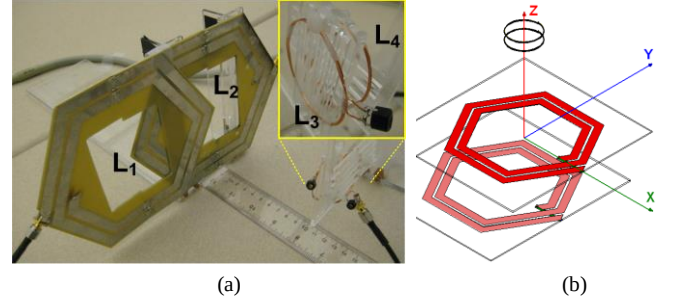


Fig. 9. (a) Experimental setup for measuring the PTE and PDL in a 4-coil inductive link. (b) 4-coil inductive link model in the HFSS. Coil specifications are listed in Table I.

efficiency has some dependency on the reflected impedance onto the Tx side, which changes with d_{23} . Nonetheless they are close to the other values.

It can be seen in Fig. 10a that the 3- and 4-coil inductive link PTEs (37% and 35%, respectively) are significantly higher than the PTE of the 2-coil link (15%) at $d_{23} = 12$ cm. At the same coupling distance, however, the 3-coil inductive link has achieved a PDL of 260 mW from $V_s = 1$ V, which is 1.5 and 59 times higher than the PDL of 2- and 4-coil links, respectively. Despite its high PTE, the 4-coil link has only been able to deliver 4.4 mW to the load under these conditions, which may not be sufficient for most applications. It thus requires a much higher V_s (~7.7 times in this case) to become comparable to its 3-coil counterpart. It should be mentioned that multi-coil links are more sensitive to resonance frequency variations due to employed high-Q intermediate coils.

The R_s value, which depends on the PA design, plays an important role in optimization of the overall power efficiency from the energy source to the load. Other key parameters that affect the PA design are the power required by the load (P_L), source voltage (V_s), supply voltage (V_{DD}), transistors breakdown voltage, and safety limits for the IMD applications [14]. The available power from source, P_{av} , can be expressed as $V_s^2/8R_s$, which implies that large V_s or small R_s are desired when P_L is large. In a class-E PA, zero-voltage-switching allows for high power efficiency with peak voltages across the coil and PA transistor that are 1.07 and 3.56 times V_{DD} , respectively [35]. Therefore, when the application involves large P_L in the order of 100s of mW, R_s should be reduced to levels well below 1Ω for the PA to provide sufficient P_{av} at

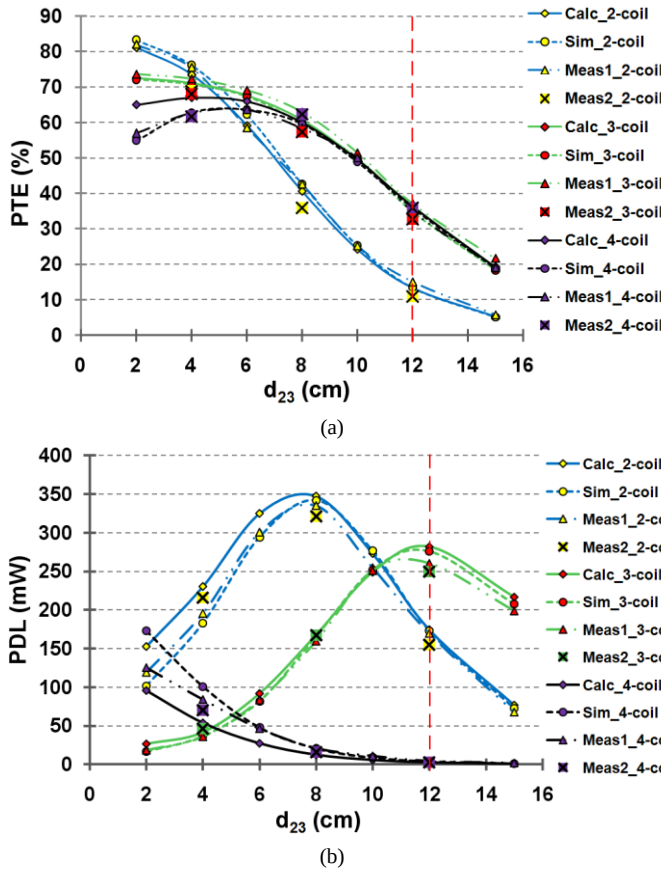


Fig. 10. Comparison between measured, simulated (HFSS), and calculated values of the (a) PTE and (b) PDL vs. d_{23} for 2-, 3-, and 4-coil inductive links specified in Table I ($V_s = 1$ V).

reasonable V_s and V_{DD} .

To compare the effects of R_s on 2-, 3- and 4-coil inductive links optimization, including the PA losses, we have optimized our design example in Table I for different values of R_s from 0.1 to 5 Ω based on the design procedure in [33]. It can be seen in Fig. 11 that the 4-coil link maintains its high PTE even at large R_s values due to its large reflected impedance, $R_{ref} \gg R_s$, at the cost of very small PDL. On the other hand, for R_s values below 1 Ω , the 3-coil link offers almost the same PTE, while providing much higher PDL. Therefore, we can conclude that for the applications that require small amounts of PDL in the order of 10s of mW, a 4-coil inductive link with a weak driver provides the highest PTE while keeping V_s within reasonable range. Because utilizing large transistors in this case to reduce R_s results in increased dynamic switching losses [24].

V. CONCLUSIONS

We have extended conventional 2-coil inductive link equations to a multi-coil arrangement to provide a platform for the analysis and design of the state-of-the-art power transmission inductive links. We have shown that both circuit and couple-mode theories result in the same set of equations for the PTE of multi-coil links. We have shown that the 3-coil inductive links can significantly improve the PTE and PDL,

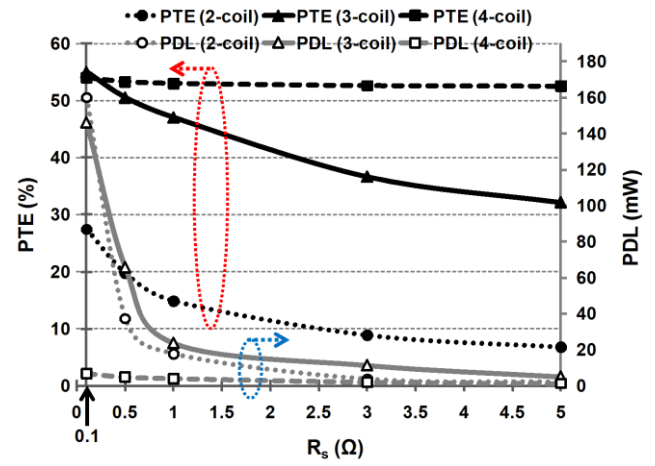


Fig. 11. Calculated PTE and PDL vs. source resistance, R_s , for 2-, 3-, and 4-coil inductive links ($V_s = 1$ V, $d_{23} = 12$ cm).

particularly at large coupling distances by transforming any arbitrary load impedance to the optimal impedance needed at the input of the inductive link. The coupling between L_3 and L_4 on the Rx side (k_{34}), provides designers with a new degree of freedom for impedance transformation, which was not feasible in 2-coil links. We showed that 4-coil inductive links transform the load impedance to a very high reflected resistance across the driver coil, which limit the available power from source and drastically reduce PDL, particularly at large coupling distances. Furthermore, a set of 2-, 3-, and 4-coil links was optimized, modeled in HFSS, and fabricated using magnet wires and PCB. Measured results at 12 cm coupling distance showed PTE of 15%, 37%, and 35% for 2-, 3-, and 4-coil links, respectively. The 3-coil link, however, achieved a PDL of 1.5 and 59 times larger than its 2- and 4-coil counterparts, respectively.

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